Forcing-based cut-elimination for Gentzen-style intuitionistic sequent calculus

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Abstract. We give a simple intuitionistic completeness proof of Kripke semantics with constant domain for intuitionistic logic with implication and universal quantification. We use a cut-free intuitionistic sequent calculus as formal system and by combining soundness with completeness, we obtain an executable cut-elimination procedure. The proof, which has been formalised in the Coq proof assistant, easily extends to the case of the absurdity connective using Kripke models with exploding nodes.

Keywords: Intuitionistic Gentzen-style sequent calculus, Kripke semantics, completeness, cut-elimination

1 Introduction

The intuitionistic completeness proofs for intuitionistic full first-order predicate logic given by Veldman [1] and Friedman [2, Chapter 13] use nonstandard Kripke model and Beth model, respectively (the false formula may be forced at some nodes). Both the proof of Veldman and that of Friedman work by building a model made of infinite contexts. Especially, they had to deal with language extensions and work with spreads in order to meet some closure conditions for disjunction and existential quantification:

- $\Gamma \vdash A \vee B$ implies $\Gamma \vdash A$ or $\Gamma \vdash B$.
- $\Gamma \vdash \exists x\, A(x)$ implies $\Gamma \vdash A(c)$ for some constant c.

Note however that this is not only the case for intuitionistic proofs, but also the case for a classical, Henkin-type proof given in Troelstra and van Dalen [2, Chapter 2].

On the other hand, C. Coquand [3] shows that an intuitionistic proof for intuitionistic propositional logic with implication as a sole logical symbol can be obtained in a much simpler way by building a universal model made of finite contexts of formulae. She gave a mechanised proof of the completeness proof and even got a cut-elimination proof by using some interpreter and inversion functions, a method called "normalisation by evaluation" in general, cf. [4]. They correspond to the soundness and the completeness, respectively, of the propositional logic with implication as sole connective w.r.t. Kripke semantics. The completeness result there is strong in two ways: in the traditional sense that it holds in arbitrary contexts (see [2]) and in a sense (due to Okada [5]) that it builds normal proofs. In this paper, we extend C. Coquand's idea to the intuitionistic first-order predicate logic with implication and universal quantification as logical symbols.

The predicate system used here is a Gentzen-style sequent calculus. The advantage is that the notion of normal proofs is easy to define: one just has to remove the cut rule. More precisely, the calculus we consider is the intuitionistic restriction LJT of a sequent calculus named LKT that Danos *et al* [6] derived from an analytical decomposition of Gentzen's LK within Girard's Linear Logic [7]. LKT is a constrained variant of LK. Its main property is the bijective correspondence between its set of cut-free proofs and the set of normal forms[3] of classical natural deduction [8]. LJT itself is a constrained variant of LJ and its main property is the bijective correspondence between its set of cut-free proofs and the set of normal forms of natural deduction [9,10]. By choosing LJT and its cut-free variant as our reference calculus, we emphasise that our completeness theorem builds not any arbitrary proofs but cut-free ones in a subset of LJ which bijectively maps to normal natural deduction proofs (and hence to normal λ-terms).

We show that the strong completeness holds both for the system with or without $(\bot_i)$. In case of with $(\bot_i)$, we adopt Veldman's *modified* Kripke semantics. Both proofs are intuitionistic and almost the same. Therefore, we can get a very simple cut-elimination proofs as a by-product at the end. A main difference compared with Veldman's or Friedman's proof is that we deal with contexts made of formulae, not just of sentences, therefore we need to handle substitutions.

The Kripke models we are considering are Kripke models with constant domain, i.e. with the domain function D being the same for every world:

$$D = D(w) \text{ for all } w.$$

As a consequence, in the case of universal quantification, considering of all possible future worlds is not necessary and the following simpler definition

$$w \Vdash \forall x\, A(x) \text{ iff, for all } d \in D,\ w \Vdash A(d).$$

gets equivalent, by monotonicity of forcing and invariance of the domain, to the standard definition:

$$w \Vdash \forall x\, A(x) \text{ iff, for all } w' \geq w \text{ and for all } d \in D,\ w' \Vdash A(d).$$

[3] To be precise: normal forms along a call-by-name reduction semantics.

This paper is organised as follows. We first prove the soundness of the Kripke semantics with respect to cut-free LJT. Then give a intuitionistic strong completeness proof which results in the intuitionistic completeness proof. Finally, an intuitionistic cut-elimination process is described.

2 The sequent calculus LJT

Let $\mathcal{L} = \mathcal{L}(\mathbb{C}, \mathbb{F}, \mathbb{P})$ be a first-order language with an infinite set $\mathbb{C}$ of individual constants, among them a distinguished constant c_0, a non-empty set $\mathbb{F}$ of functions, and a non-empty set $\mathbb{P}$ of predicates. The logical symbols are the implication $\rightarrow$ and the universal quantification $\forall$. We assume furthermore that there are countably many free variables. Terms, formulae and sentences are defined in the usual way. We follow the convention that $A(x)$ denotes the formula A where the variable x might appear. Any two formulae are considered identical when they are different only in names for bound variables. A variable or a constant is called fresh in a formula A or a context Γ when it does not occur free or at all, respectively.

Definition 1 (Simultaneous substitutions). *Let ρ be a function from finite set of variables to the set of terms.*

1. *Given a term t, $t[.\backslash\rho]$ is inductively defined:*
   - $x[.\backslash\rho] = \begin{cases} \rho(x) & \text{if } x \in \mathsf{dom}(\rho), \\ x & \text{otherwise.} \end{cases}$
   - $c[.\backslash\rho] = c$ *for any* $c \in \mathbb{C}$.
   - $(f\, t_1 \cdots t_n)[.\backslash\rho] = f\,(t_1[.\backslash\rho]) \cdots (t_n[.\backslash\rho])$.
2. *Given a formula A, $A[.\backslash\rho]$ is inductively defined:*
   - $(P\, t_1 \cdots t_n)[.\backslash\rho] = P\,(t_1[.\backslash\rho]) \cdots (t_n[.\backslash\rho])$.
   - $(A \rightarrow B)[.\backslash\rho] = (A[.\backslash\rho] \rightarrow (B[.\backslash\rho])$.
   - $(\forall x\, A)[.\backslash\rho] = \forall x\,(A[.\backslash\rho^{-x}])$.

   Here ρ^{-x} denotes the function obtained from ρ with $\mathsf{dom}(\rho^{-x}) = \mathsf{dom}(\rho)\backslash\{x\}$, i.e., if $y \in \mathsf{dom}(\rho)$ and $x \neq y$, then $\rho^{-x}(y) = \rho(y)$ and undefined otherwise. We also take care of variable capture by changing bound variables when necessary.

Given a formula $A(x)$, we use $A_x(t)$ or $A[x\backslash t]$ for $A[.\backslash\rho]$ where $\rho = \{(x,t)\}$. We also consider substitution of a term t for a constant c in a similar way and use the notation $A_c(t)$.

The Gentzen-style sequence calculus LJT is obtained from the intuitionistic sequent calculus LJ by restricting the use of the left introduction rules of the implication and the universal quantification. See Table 1 for the cut-free fragment. In that way, one can get a one-to-one correspondence between cut-free proofs in LJT and normal terms in λ-style calculus.

In LJT, a sequent has one of the forms $\Gamma\,; A \vdash C$ or $\Gamma \vdash C$, where Γ is a list of formulae. That is, the location of a formula occurring multiple times is important. The right side of ";" in the antecedence is called stoup . $\Gamma, \Gamma', \Delta, ...$

$$\frac{}{\Gamma\,;A \vdash A}\ (Ax) \qquad \frac{\Gamma\,;A \vdash C \quad A \in \Gamma}{\Gamma \vdash C}\ (Contr)$$

$$\frac{\Gamma \vdash A \quad \Gamma\,;B \vdash C}{\Gamma\,;A \to B \vdash C}\ (\to_\ell) \qquad \frac{A, \Gamma \vdash B}{\Gamma \vdash A \to B}\ (\to_r)$$

$$\frac{\Gamma\,;A_x(t) \vdash C}{\Gamma\,;\forall x\, A(x) \vdash C}\ (\forall_\ell) \qquad \frac{\Gamma \vdash A(x) \quad x \text{ fresh in } \Gamma}{\Gamma \vdash \forall x\, A}\ (\forall_r)$$

Table 1. Cut-free LJT

vary over lists of formulae. We write $A \in \Gamma$ when A occurs in Γ. $\Gamma \sqsubseteq \Delta$ denotes that, for all A, $A \in \Gamma$ implies $A \in \Delta$. $\Gamma_c(t)$ is obtained from Γ by replacing each formula A with $A_c(t)$.

Lemma 2 (Weakening). *Let A, C be formulae and Γ, Γ' two contexts such that $\Gamma \sqsubseteq \Gamma'$.*

1. *$\Gamma \vdash C$ implies $\Gamma' \vdash C$.*
2. *$\Gamma\,;A \vdash C$ implies $\Gamma'\,;A \vdash C$.*

Proof. One can easily prove both claims by a simultaneous induction on the deduction. □

Lemma 3. *Let Γ be a context, A, C formulae, and c a constant.*

1. *$\Gamma \vdash C$ implies $\Gamma_c(y) \vdash C_c(y)$ for any variable y which is not bound in Γ, A.*
2. *$\Gamma\,;A \vdash C$ implies $\Gamma_c(y)\,;A_c(y) \vdash C_c(y)$ for any variable y which is not bound in Γ, A, C.*

Proof. By a simple simultaneous induction on deduction. □

The following lemma says that a fresh constant is as good as a fresh variable and will play an important role in the proof of the strong completeness.

Lemma 4. *Given a context Γ, a formula $A(x)$, and a constant c fresh in Γ and $A(x)$, $\Gamma \vdash A_x(c)$ implies $\Gamma \vdash A_x(y)$ for any variable y which is not bound in Γ, A.*

Proof. It follows directly from the lemma just before. □

3 Kripke semantics

Kripke semantics was created in the late 1950s and early 1960s by Saul Kripke [11,12]. It was first made for modal logic, and later adapted to intuitionistic logic and other non-classical systems. In this section we discuss Kripke models for the first-order predicate logic with implication and universal quantification as sole logical symbols and their connection with intuitionistic validity.

Definition 5. *A Kripke model is a quadruple $\mathcal{K} = (\mathcal{W}, \leq, \Vdash, \mathcal{D}, V)$, $\mathcal{W}$ inhabited, such that*

1. *$(\mathcal{W}, \leq)$ is a partially ordered set.*
2. *$\mathcal{D}$ is an inhabited set, called the* domain *of $\mathcal{K}$.*
3. *Let the language be extended with constant symbols for each element of $\mathcal{D}$. Then $\Vdash$ is a relation between $\mathcal{W}$ and the set of prime sentences in the extended language such that*

$$(w \leq w' \wedge w \Vdash P\, d_1 \cdots d_n) \quad \Rightarrow \quad w' \Vdash P\, d_1 \cdots d_n$$

   where $w, w' \in \mathcal{W}$, $P \in \mathbb{P}$, $d_1, ..., d_n \in \mathcal{W}$, and $n = arity(P)$.
4. *V is a function such that*
   - *$V(c) \in \mathcal{D}$ for all $c \in \mathbb{C}$.*
   - *$V(f) : \mathcal{D}^{arity(f)} \to \mathcal{D}$ for all $f \in \mathbb{F}$.*

An association ρ based on $\mathcal{K}$ is a function from a finite set of variables to $\mathcal{D}$. Given an association ρ, each term has an interpretation in $\mathcal{D}$:

- $x\,[\rho] = \begin{cases} \rho(x) & \textit{if } x \in \mathsf{dom}(\rho), \\ V(c_0) & \textit{otherwise.} \end{cases}$
- $c\,[\rho] = V(c)$ *for any* $c \in \mathbb{C}$.
- $(f\, t_1 \cdots t_\ell)\,[\rho] = V(f)(t_1\,[\rho]) \cdots (t_\ell\,[\rho])$.

The forcing relation is then inductively extended by the forthcoming clauses to all $\mathcal{L}$-formulae.

- $w \Vdash (P\, t_1 \cdots t_n)[\rho]$ *iff* $w \Vdash P\,(t_1\,[\rho]) \cdots (t_n\,[\rho])$.
- $w \Vdash (A \to B)[\rho]$ *iff, for all* $w' \geq w$, *if* $w' \Vdash A\,[\rho]$, *then* $w' \Vdash B\,[\rho]$.
- $w \Vdash (\forall x\, A)[\rho]$ *iff, for all* $d \in D$, $w \Vdash A\,[\rho(x \mapsto d)]$.

Here $\rho(x \mapsto d)$ denotes the association ρ' such that $\rho'(y) = \rho(y)$ if $y \neq x$ and $\rho'(x) = d$. The definition of forcing is extended to contexts as follows:

$$w \Vdash \Gamma\,[\rho] \textit{ iff } w \Vdash A\,[\rho] \textit{ for all } A \in \Gamma,$$

Remark 6. There are two points to be mentioned.

1. The forcing relation is upward monotone, i.e., if $w \leq w'$ and $w \Vdash A\,[\rho]$, then $w' \Vdash A\,[\rho]$.
2. As mentioned in the introduction, the forcing definition at the universal quantification case is much simpler than the usual definition, where the Domain depends on worlds:

$$w \Vdash (\forall x\, A(x))[\rho] \text{ iff, for all } w' \geq w \text{ and } d \in D(w),\ w' \Vdash A\,[\rho(x \to d)].$$

   Indeed, they are "functionally equivalent" in the sense that soundness and completeness hold in both cases.

We consider a formulation of Kripke semantics for sequent calculus built on two kinds of judgements $\Gamma \vdash C$ and $\Gamma\,; A \vdash C$.

Theorem 7 (Soundness). *We have the following soundness.*

1. *if $\Gamma \vdash C$ then, for all w and ρ, $w \Vdash \Gamma\,[\rho]$ implies $w \Vdash C\,[\rho]$.*
2. *if $\Gamma\,; A \vdash C$ then, for all w and ρ, $(w \Vdash \Gamma\,[\rho]$ and $w \Vdash A\,[\rho])$ implies $w \Vdash C\,[\rho]$.*

Proof. By a simultaneous induction on the deduction. □

4 Completeness

In this section we present a constructive completeness proof by constructing a simple universal model. First we construct a universal model for which a strong completeness holds.

Definition 8 (Universal Kripke model). *The universal Kripke model $\mathcal{U} = (\mathcal{W}_u, \sqsubseteq, \Vdash_u, \mathcal{D}_u, V_u)$ is defined as follows:*

- *$\mathcal{W}_u$ is the set of all contexts.*
- *$\sqsubseteq$ denotes the sub-context relation, i.e., $\Gamma \sqsubseteq \Gamma'$ holds when, for all $A \in \Gamma$, $A \in \Gamma'$.*
- *$\mathcal{D}_u$ consists of all closed terms.*
- *$V_u(c) = c$ for all $c \in \mathbb{C}$, and $V_u(f)(t_1, ..., t_\ell) = f\,t_1 \cdots t_\ell$ for all $f \in \mathbb{F}$ and $t_1, ..., t_\ell \in \mathcal{D}_u^{arity(f)}$*
- *$\Gamma \Vdash P\,t_1 \cdots t_\ell$ if $\Gamma \vdash P\,t_1 \cdots t_\ell$. It is obvious that $\Gamma \sqsubseteq \Gamma'$ and $\Gamma \vdash P\,t_1 \cdots t_\ell$ imply $\Gamma' \vdash t_1 \cdots t_\ell$.*

Theorem 9 (Strong Completeness). *Let Γ be a context of sentences, A a formula, and ρ an association based on $\mathcal{K}_u$ such that $FV(A) \subseteq \mathsf{dom}(\rho)$. Then*

1. *If $\Gamma \Vdash A\,[\rho]$ then $\Gamma \vdash A[.\backslash\rho]$.*
2. *If, for all formula C and context Γ' such that $\Gamma \sqsubseteq \Gamma'$, $\Gamma'\,; A[.\backslash\rho] \vdash C$ implies $\Gamma' \vdash C$, then it holds that $\Gamma \Vdash A\,[\rho]$.*

Proof. Given the assumptions we prove both claims by a simultaneous induction on the complexity of A. Note first that $t\,[\rho] = t[.\backslash\rho]$ for any term t occurring in A since $FV(A) \subseteq \mathsf{dom}(\rho)$.

1. case: A is a prime formula. Then the first claim is obvious. For the second claim take just $\Gamma' := \Gamma$ and $C := A[.\backslash\rho]$.

2. case: $A = A_1 \to A_2$.

- Assume $\Gamma \Vdash (A_1 \to A_2)\,[\rho]$. To show $\Gamma \vdash A_1[.\backslash\rho] \to A_2[.\backslash\rho]$, it suffices to prove that $A_1[.\backslash\rho], \Gamma \vdash A_2[.\backslash\rho]$. Note that the i.h. on A_1 for the second claim implies that $A_1[.\backslash\rho], \Gamma \Vdash A_1\,[\rho]$. Indeed, the premise of the second claim holds trivially for any Γ' such that $A_1[.\backslash\rho], \Gamma \sqsubseteq \Gamma'$. This in turn implies that $A_1[.\backslash\rho], \Gamma \Vdash A_2\,[\rho]$ by the assumption. Then the i.h. on A_2 for the first claim leads to the goal.

- Assume for all formula C and context Γ' such that $\Gamma \sqsubseteq \Gamma'$, $\Gamma' ; A_1[.\backslash\rho] \to A_2[.\backslash\rho] \vdash C$ implies $\Gamma' \vdash C$. Assume furthermore that $\Gamma \sqsubseteq \Delta$ and $\Delta \Vdash A_1\,[\rho]$. Then it remains to show $\Delta \Vdash A_2\,[\rho]$. For that we apply the i.h. on A_2 for the second claim. Let C be a formula and Δ' a context such that $\Delta \sqsubseteq \Delta'$, assume $\Delta' ; A_2[.\backslash\rho] \vdash C$. Note that $\Delta \vdash A_1[.\backslash\rho]$ by i.h. on A_1 for the first claim, hence $\Delta' \vdash A_1[.\backslash\rho]$ by the Weakening Lemma 2. By applying $(\to_\ell)$ we get $\Delta' ;(A_1 \to A_2)[.\backslash\rho] \vdash C$, so $\Delta' \vdash C$ holds by the assumption.

3. case: $A = \forall x\, B(x)$.

- Assume $\Gamma \Vdash (\forall x\, B)\,[\rho]$, i.e., for all closed term t, $\Gamma \Vdash B\,[\rho(x \mapsto t)]$. To show $\Gamma \vdash \forall x\,(B[.\backslash\rho^{-x}])$, we need to prove that $\Gamma \vdash (B[.\backslash\rho^{-x}])[x\backslash y]$ for some variable y fresh in Γ and $B[.\backslash\rho^{-x}]$. For this we show $\Gamma \vdash (B[.\backslash\rho^{-x}])[x\backslash c]$ for some constant c and apply Lemma 4. Let c be a constant fresh in Γ and $B[.\backslash\rho^{-x}]$. Note first that

$$(B[.\backslash\rho^{-x}])[x\backslash c] = B[.\backslash\rho^{-x}(x \mapsto c)] = B[.\backslash\rho(x \mapsto c)]$$

since the values of ρ are closed terms. However, $\Gamma \vdash B[.\backslash\rho(x \mapsto c)]$ follows from the i.h. on B for the first claim.[4]
- Assume for all formula C and context Γ' such that $\Gamma \sqsubseteq \Gamma'$, it holds that $\Gamma' ; \forall x\,(B[.\backslash\rho^{-x}]) \vdash C$ implies $\Gamma' \vdash C$. Given a closed term t, we have to show that $\Gamma \Vdash B\,[\rho(x \mapsto t)]$. In order to apply the i.h. on B for the second claim assume furthermore that a formula C and a context Γ' are given such that $\Gamma \sqsubseteq \Gamma'$ and $\Gamma' ; B[.\backslash\rho(x \mapsto t)] \vdash C$. Note that $B[.\backslash\rho(x \mapsto t)] = B[.\backslash\rho^{-x}][x\backslash t]$ since only closed terms are substituted. Therefore, it holds that $\Gamma' ; \forall x\,(B[.\backslash\rho^{-x}]) \vdash C$. Then by the main assumption, $\Gamma' \vdash C$. The i.h. implies $\Gamma \Vdash B\,[\rho(x \mapsto t)]$. ☐

Corollary 10. *For any context Γ of sentences, $\Gamma \Vdash \Gamma$.*

Proof. The second claim of the strong completeness and the rule $(Unload)$ implies that $\Gamma \Vdash A$ for any $A \in \Gamma$. ☐

Theorem 11 (Completeness). *Let Γ be a context of sentences and A a sentence. If for all Kripke model $\mathcal{K}$ and a world w in $\mathcal{K}$, $w \Vdash \Gamma$ implies $w \Vdash A$, then $\Gamma \vdash A$.*

Proof. It follows from the Strong Completeness and the fact that $\Gamma \Vdash \Gamma$, i.e., $\Gamma \vdash A$ iff $\Gamma \Vdash A$. ☐

Remark 12. The completeness above easily extends to the case of the absurdity connective

$$\frac{\Gamma \vdash \bot}{\Gamma \vdash C}\ (\bot_i)$$

using modified Kripke models with exploding nodes à la Veldman. A modified Kripke model $\mathcal{K} = (\mathcal{W}, \leq, \Vdash, \mathcal{D}, V)$ is defined as the (unmodified) Kripke model, but with one change:

[4] One can see here that we don't need any quantification in the definition of $w \Vdash \forall x\, A(x)$.

– $w \Vdash (P\, t_1 \cdots t_n)[\rho]$ iff $(w \Vdash P\,(t_1\,[\rho]) \cdots (t_n\,[\rho])$ or $w \Vdash \bot)$.

The universal Kripke model is defined in the same way as before, but with the following additional clause:

– $\Gamma \Vdash \bot$ iff $\Gamma \vdash \bot$.

Then nothing new is involved in the proof of completeness, the construction proceeds as before. Note only that $\Gamma \vdash \bot$ implies $\Gamma \vdash A$ for any formula A.

On the other hand, if we include the absurdity rule and want to stick to the (unmodified) Kripke semantics, we have to give up constructiveness in the proof above. This is because we have to deal only with consistent context Γ, i.e., $\Gamma \nvdash \bot$. However, it is in general undecidable to check if a context is consistent or not. In the implication case of the strong completeness, we had to make case distinction between $A_1[.\backslash\rho], \Gamma \vdash \bot$ or not. This maybe is not so surprising because, in the full first-order predicate logic, an intuitionistic completeness proof entails Markov's Principle, a non-intuitionistic principle, see Kreisel [13].

5 Cut admissibility

In this section we consider only sentences and contexts of sentences. Then the cut-rule is admissible with/without $(\bot_i)$.

Lemma 13 (Cut admissibility). *$\Gamma \vdash A$ and $\Gamma\,; A \vdash B$ imply $\Gamma \vdash B$.*

Proof. By the Strong Completeness it suffices to show $\Gamma \Vdash B$. But this follows from the Soundness applied on the two assumptions. Note also that $\vdash$ denotes a cut-free system. □

6 Conclusion

We extended C. Coquand's proof of soundness and completeness for implicative natural deduction w.r.t. Kripke semantics [3] to the case of predicate logic with implication and universal quantification. We could show that omitting disjunction and existential quantification from the intuitionistic first-order predicate logic results, as it was the case for C. Coquand, in a significantly simple, intuitionistic completeness proof with respect to (also simplified) Kripke semantics.

The fact that all of the proofs given in this paper are intuitionistic has been verified in the proof assistant Coq, cf. [14]. Indeed, the whole work is formalised, so that we can get a mechanical process producing cut-free proofs. The formalisation is performed using cofinite quantification for fresh variables and a locally named approach with two kinds of names for variables, one for free variables and the other for binders. The formalisation is publicly available at `http://pauillac.inria.fr/~herbelin/code/kripke`.

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